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Method for Predicting Creep in Tension and Compression from Bending Tests

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It is shown that tensile and compressive creep behavior may be deduced from deflection measurements in creep bending tests on beams of trapezoidal cross section. In the analysis it is assumed that creep strains are proportional to stress and are large relative to elastic strains.

IN APPLYING compressive creep data to predict creep in tension or in analyzing bending creep data, it is usually assumed that tensile and compressive creep are identical. Unfortunately, it is hardly possible to evaluate this assumption for ceramics because of the lack of experimental evidence, but for metals and organic polymers a number of studies have shown pronounced differences between tensile and compressive creep.

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Table I. Summary of Creep Tests on Polycrystalline Ceramics

Material and reference	Type of test*	Test temp. (°C)	Range of stress (psi)	Approx. grain diameter (μ)	Exponent n in $\epsilon \sim \sigma^n$
Aluminum oxide (Ref. 1)	B3	1400–1800	100–15,000 at 1800°C	7–34	1
Aluminum oxide (Ref. 2)	B4	1600–1800	100–2000	3–13	1
				50–100	4
Magnesium oxide (Ref. 3)	B4	1180–1250	2000–2800 at 1250°C	1–3	1
			1600–2600 at 1180°C		
Beryllium oxide (Ref. 4)	C	1370–1540	1500–6000	7.5–10	1
Beryllium oxide (Ref. 5)	C	1200	1000–6000		1
			6000–10,000	3–89	>1†

* B3 = three-point bending, B4 = four-point bending, and C = compression.
† Generally, tests at higher stresses appear to have been made on specimens of larger grain size.

If the outer fiber strains are measured in creep bending tests, it is possible with certain assumptions to obtain tensile and compressive creep data, and, conversely, if tension and compression creep data are available, it is possible to predict creep in bending. At the very high temperatures that are of interest in the creep testing of ceramics, however, both tension testing and the measurement of surface strain in a bending test present great difficulty. With these limitations in mind, the purpose of this paper is to outline a method by which tensile and compressive creep data may be obtained from bending deflection measurements on beams of trapezoidal cross section.

The creep strains produced by constant stress tension and compression will be taken as $\epsilon_t = \sigma F$, $\epsilon_c = \beta \sigma F$, where σ is stress, β is the ratio of compressive to tensile creep at a given stress, and F is an arbitrary function of time. That the assumption of linear stress dependence is reasonable for fine-grained polycrystalline ceramics can be seen from Table I which summarizes a number of recently reported creep tests. By contrast, creep tests on single crystals of aluminum oxide at 1000°C showed creep proportional to the sixth power of stress[6] and tests on magnesium oxide at 1450° to 1700°C showed creep proportional to the fourth power of stress.[7] Similarly, in metals the dependence of creep strain on stress is normally nonlinear, whereas organic polymers usually show a linear stress dependence unless the strains become large. The assumption that tensile and compressive creep strain show the same time dependence is probably reasonable but, in any event, it may be checked from the test results. In addition, to simplify the analysis, it will be assumed that elastic strains are negligible compared to creep strains. Although this assumption limits the generality of the analysis, it permits a closed form analytical solution which should be adequate in many cases.

Since plane cross sections before bending remain plane during pure bending, the strain ϵ_y at distance y from the neutral axis may be written as $\epsilon_y = \epsilon_{H_c}(y/H_c)$ where ϵ_{H_c} is the strain at the outer fiber, distance H_c from the neutral axis. Other details of nomenclature are shown in Fig. 1. The preceding equation may be rewritten in terms of stress as: $\sigma_y = \sigma_{H_c}(y/H_c)$ for compression and $\sigma_y = \beta\sigma_{H_c}(y/H_c)$ for tension. The condition of axial force equilibrium for the section shown in Fig. 1(*a*) is

$$\int_{-H_t}^{H_c} \sigma_y \, b \, dy = 0$$

Putting $h_c = H_c/H$, $h_t = H_t/H$, and making use of $h_c + h_t = 1$ leads, after manipulation, to

$$\frac{1}{3}(1-\beta)\left(\frac{b_2}{b_1} - 1\right) h_c^3 + (1-\beta) h_c^2 + \beta\left(\frac{b_2}{b_1} + 1\right) h_c - \frac{\beta}{3} 2\left(\frac{b_2}{b_1} + 1\right) = 0 \quad (1)$$

This equation is shown graphically in Fig. 2 and makes it possible to locate the neutral axis for various values of β and b_2/b_1. By symmetry, the same graph applies also to the inverted section shown in Fig. 1(*b*) if h_c is replaced by $h'_t = H'_t/H$ and β is replaced by $1/\beta$.

The condition of moment equilibrium for the section shown in Fig. 1(*a*) is

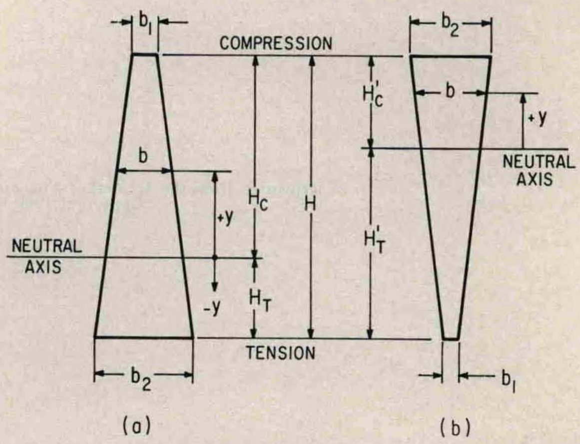

Fig. 1. Nomenclature used to describe beams of trapezoidal cross section.

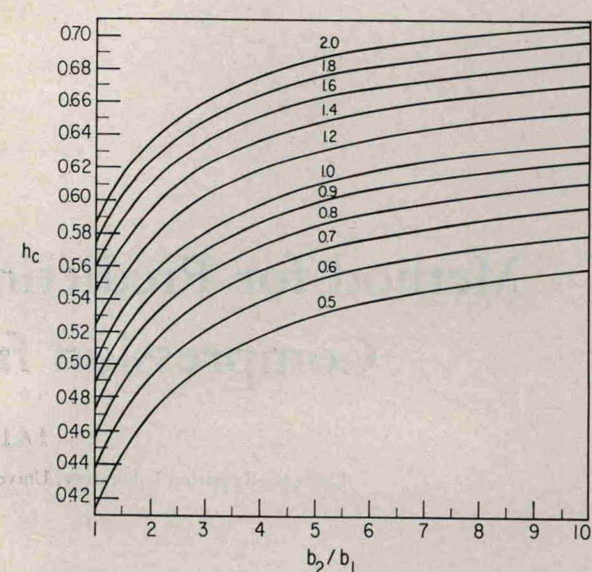

Fig. 2. Distance h_c as function of b_2/b_1 for several values of β.

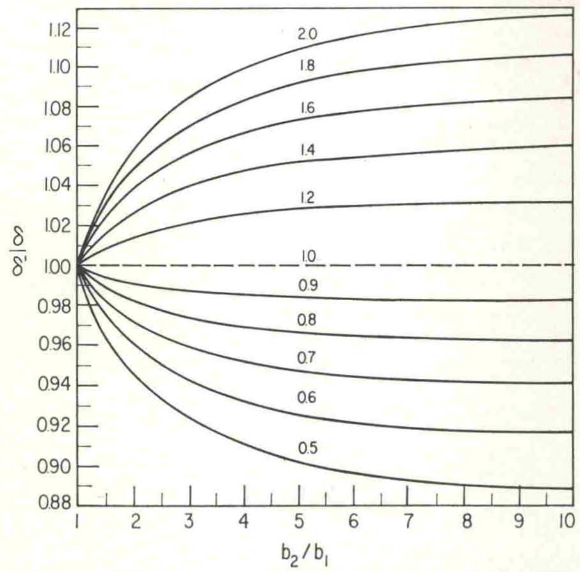

Fig. 3. Deflection ratio as function of b_2/b_1 for several values of β.

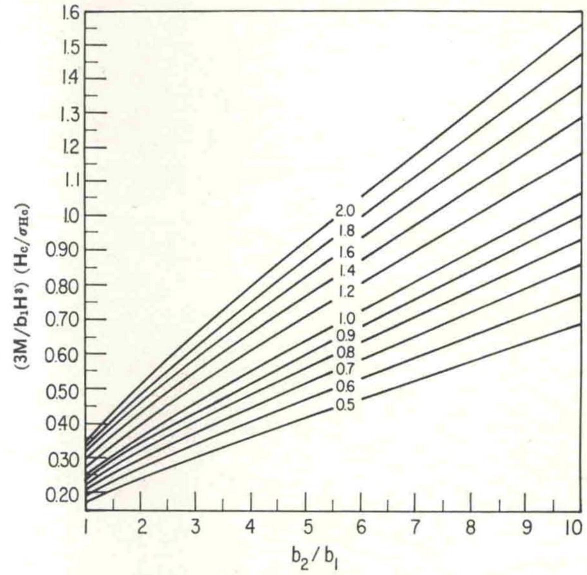

Fig. 4. Quantity used to obtain creep constants as a function of b_2/b_1 for several values of β.

$$M = \int_{-H_t}^{H_c} \sigma_y y b \, dy$$

After substitution for σ and b and integration this becomes

$$\frac{3}{b_1 H^3}\frac{MH_c}{\sigma_{H_c}} = h_c^3 + \frac{1}{4}\left(\frac{b_2}{b_1} - 1\right) h_c^4 + \beta\left[h_t^3 + \left(\frac{b_2}{b_1} - 1\right) h_c h_t^3 + \frac{3}{4}\left(\frac{b_2}{b_1} - 1\right) h_t^4\right] \quad (2)$$

Similarly for this section shown in Fig. 1(*b*),

$$\frac{3}{b_1 H^3}\frac{MH'_c}{\sigma'_{H_c}} = \beta\left[h'^3_t + \frac{1}{4}\left(\frac{b_2}{b_1} - 1\right) h'^4_t\right] + h'^3_c + \left(\frac{b_2}{b_1} - 1\right) h'^3_c h'_t + \frac{3}{4}\left(\frac{b_2}{b_1} - 1\right) h'^4_c \quad (3)$$

With the preceding equations, the bending stresses may be determined for a given cross section and bending moment if the parameter β is known. As β has to be determined, however, the equations must be used in somewhat different form.

In pure bending, the radius of curvature R of the neutral axis at a given time is related to the surface strain by $1/R = \epsilon_{H_c}/H_c = \beta\sigma_{H_c}F/H_c$ and similarly $1/R' = \beta\sigma_{H'_c}F/H'_c$. Hence the ratio of the two curvatures at a given time $1/R \div 1/R'$ is merely the right-hand side of Eq. (3) divided by the right-hand side of Eq. (2). The curvature is given by $1/R = d^2y/dx^2/[1 + (dy/dx)^2]^{3/2}$, where y is bending deflection and x is distance along the beam. In most practical cases the slope dy/dx is small enough for this to be written as $1/R = d^2y/dx^2$ and bending deflections will then be directly proportional to $1/R$. Thus, if the beams with cross sections shown in Fig. 1 are subjected to the same bending moments the ratio of deflections δ/δ' at corresponding locations along the beam at a given time are given by

$$\delta/\delta' = \frac{1/R}{1/R'} = \frac{\text{Right-hand side Eq. (3)}}{\text{Right-hand side Eq. (2)}}$$

This relation is shown in Fig. 3 for a range of values of β and b_2/b_1. By observing the ratio at several times, for which creep strains are large relative to elastic strains, the ratio β may be found. If β varies greatly with time the preceding analysis is not applicable, whereas if $\beta = 1$, tension and compression creep are equivalent and the analysis of bending tests presents no difficulty. If $\beta \neq 1$, the next step is to determine the individual tension or compression creep data. This may be done by using Eq. (2) to find σ_{H_c} for a given bending moment. Measurements of curvature as a function of time (obtained from bending deflection data) then make it possible to determine F from $F = (1/R)(H_c/\sigma_{H_c})(1/\beta)$. For this purpose Fig. 4 shows $(3M/b_1H^3)(H_c/\sigma_{H_c})$ for various values of β and b_2/b_1.

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